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A STUDY OF THE MERIDIONAL CONVERGENCE  
OF ANGULAR MOMENTUM AT 500 MBS  
IN SELECTED LATITUDE BELTS

FREDERICK F. DUGGAN, JR.

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MOMENTUM AT 500 MBS IN SELECTED LATITUDE BELTS

by

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Lieutenant, United States Navy

Submitted in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

IN

METEOROLOGY

United States Naval Postgraduate School

Monterey, California

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A STUDY OF THE MERIDIONAL CONVERGENCE OF ANGULAR  
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Frederick F. Duggan, Jr.

This work is accepted as fulfilling the thesis requirements for degree

of

MASTER OF SCIENCE

in

METEOROLOGY

from the

United States Naval Postgraduate School



## ABSTRACT

At a given pressure level, the meridional convergence of angular momentum may be expressed in terms of the geostrophic wind components, and, in turn, by the Fourier analysis of the contour field. The contribution to the convergence of angular momentum at 500 mbs by waves of numbers 1 through 10 is computed for three ten-degree latitude belts for the period 1-20 January 1948.

An attempt to correlate this convergence with the 24-hour change in zonal geostrophic wind speed for the 10-degree latitude belt is made.

The writer is deeply indebted to Professor F. L. Martin of the United States Naval Postgraduate School for his suggestion of the topic and his continued help throughout the investigation and during the preparation of this paper.



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# LIST OF SYMBOLS USED

Symbol	Definition
$M$	absolute angular momentum
$u$	eastward component of wind
$v$	northward component of wind
$\Omega$	angular velocity of earth
$a$	radius of earth
$\phi$	latitude
$\alpha$	specific volume of air
$F_x$	horizontal friction force in direction
$r$	$a \cos \phi$ , radius of latitude circle at latitude $\phi$
$V$	volume of atmosphere in polar cap
$M$	transport of momentum across latitude wall
$P$	pressure torque across mountain ranges
$F$	torque due to friction force of the earth on the atmosphere
$\underline{n}$	unit normal directed outward from surface
$\sigma$	surface of volume of atmosphere in polar cap



Symbol	Definition
$\underline{V}$	vector wind
$\underline{V}_g$	geostrophic wind vector
$u_g$	eastward component of $\underline{V}_g$
$v_g$	northward component of $\underline{V}_g$
$\lambda$	eastward angular measurement from prime meridian
$g$	acceleration due to gravity
$n$	wave number
$i$	$\sqrt{-1}$
$V$	Fourier transform of $v$
$U$	Fourier transform of $u$
$f$	coriolis parameter
$z$	contour height
$A$	Fourier transform of $z$
$C$	amplitude of wave



Symbol	Definition
$\psi$	phase angle measured in degrees of longitude from prime meridian
$J_M(n)$	transport of momentum due to wave number $n$
$r$	linear correlation coefficient
$S$	standard deviation of a variable





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## 1. Introduction.

Considering the mean zonal wind distribution at the earth's surface, Fig. 1, it is evident that

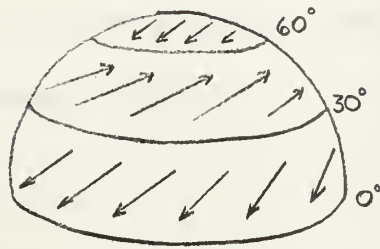


Fig. 1. Distribution of the surface winds over a longitudinally uniform earth [ 1 ].

the mid-latitude westerlies exert an eastward torque on the surface of the earth and that the easterlies exert a westward torque. Since no long-term changes in the angular velocity of the earth occur, the net integrated long-term torque exerted over the earth as a whole must be zero.

By Newton's third law, the earth exerts an equal and opposite torque on the atmosphere. Therefore in the belts of easterlies the earth exerts an eastward torque on the atmosphere, although in the polar regions the magnitude is quite small due to the short lever arm. Thus the tropical and polar atmospheres receive absolute angular momentum from the earth. In the belt of westerlies the atmosphere gives up absolute angular momentum to the earth.

Because these wind belts are maintained over a long period, the angular momentum must be transported from the tropical and polar atmospheres to the middle latitudes, where it is brought down to the earth's surface.



To make use of the available data, the geostrophic approximation may be used in the mid-latitudes to compute the meridional transfer of angular momentum.

This study is confined to the 500-mb level, and considers only the period from 1 through 20 January, 1948.

## 2. Source of Data.

The U. S. Navy Weather Research Facility at Norfolk, Virginia, made a harmonic analysis of the Historical Weather Map Series at 500 mbs from the period 1 November 1957 through 29 February 1948 [7]. Harmonics 1 through 18 were computed. The mean height, amplitude and phase angle of each harmonic were listed for every  $5^\circ$  of latitude from  $15^\circ$  N to  $70^\circ$  N.

For this study the period 1 through 20 January 1948 was selected and the study was limited to three ten-degree latitude belts between  $35^\circ$  N and  $65^\circ$  N. Mean heights at the latitudinal boundaries of each belt before, during, and after this period were used to compute zonal geostrophic winds for time-lag analysis.

## 3. Mathematical analysis.

### (a) Angular momentum changes in the polar cap.

At latitude  $\phi$ , the absolute angular momentum<sup>1</sup> of a gram of air about the polar axis, may be written (after Haltiner and Martin [1]).

$$M = (u + \Omega a \cos \phi) a \cos \phi \quad (1)$$

1. Henceforth called simply angular momentum



By Newton's second law,

$$\frac{dM}{dt} = \left( -\alpha \frac{dp}{dx} + F_x \right) r \quad (2)$$

where  $r$  is the radius of the latitude circle and  $F_x$  is the friction force.

Multiplying by  $\varphi$ , expanding, and making use of the continuity equation enables one to transform (2) to

$$\frac{\partial(\varphi M)}{\partial t} = -\nabla \cdot \varphi M \underline{V} - \frac{\partial p r}{\partial x} + F_x r \quad (3)$$

The quantity  $\varphi M$  represents angular momentum per unit volume and the quantities on the right are torques per unit volume tending to increase its value.

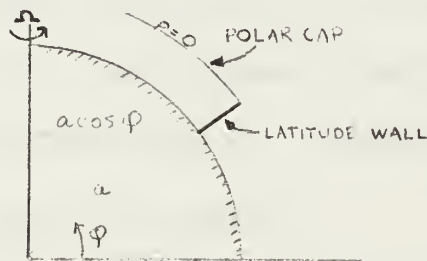


Fig. 2. Section through earth illustrating the polar cap bounded by the latitude  $\varphi$  [1].

Integrating (3) over entire volume  $\mathcal{V}$  of air poleward of latitude  $\varphi$

(Fig. 2) leads to

$$\int \frac{\partial(\varphi M)}{\partial t} \delta \mathcal{V} = -\int \nabla \cdot \varphi M \underline{V} \delta \mathcal{V} - \int \frac{\partial p r}{\partial x} \delta \mathcal{V} + \int \varphi F_x r \delta \mathcal{V} \quad (4)$$

or

$$\frac{\partial}{\partial t} \int \varphi M \delta \mathcal{V} = \mathcal{M} + \mathcal{P} + \mathcal{J} \quad (5)$$

where  $\mathcal{M}$  represents the transport of angular momentum into the polar cap. The term symbolized as  $\mathcal{P}$  stands for the corresponding term on





the right side of (4) and may be called the pressure torque. It is due to differences of pressure across mountain ranges.  $\mathcal{F}$  is the torque due to the friction force of the earth on the atmosphere. Since this study is confined to 500 mbs,  $\mathcal{P}$  and  $\mathcal{F}$  will not be further analyzed.

By definition,

$$\mathcal{M} = - \int \nabla \cdot \rho \underline{M} \underline{V} \delta \underline{V}.$$

Using the divergence theorem, with  $\underline{n}$  representing unit normal vector drawn everywhere outward to the surface,

$$\mathcal{M} = - \int \underline{n} \cdot \underline{V} \rho M \delta \sigma = \int \rho M v \delta \sigma \quad (6)$$

Here the final form of (6) results on assuming zero transports at top and bottom of the polar cap.  $v$  is the inbound or south wind and  $\delta \sigma$  is the element of area in the latitude wall,  $\delta \sigma = (a \cos \phi \delta \lambda) \delta z$ .

Expressing (6) as a double integral, one obtains

$$\mathcal{M} = \int_0^{2\pi} \int_0^\infty \rho M v a \cos \phi \delta \lambda \delta z.$$

Substituting equation (1) for  $M$ , and  $\rho \delta z = - \frac{\delta p}{g}$ , there results

$$\mathcal{M} = \frac{a^2 \cos^2 \phi}{g} \int_0^{2\pi} \int_0^{p_0} (u + \Omega a \cos \phi) v \delta \lambda \delta p. \quad (7)$$

Performing the integration with respect to longitude gives

$$\mathcal{M} = \frac{2\pi a^2 \cos^2 \phi}{g} \int_0^{p_0} \left( \underbrace{\Omega \bar{v} a \cos \phi}_{(1)} + \underbrace{\bar{u} \bar{v}}_{(2)} + \underbrace{u'v'}_{(3)} \right) \delta p, \quad (8)$$



where the bars indicate mean values computed around the entire latitude circle at latitude  $\phi$ . These three terms are called (1) the  $\Omega$ -transport term, (2) the drift-term, and (3) the eddy-flux term.

Considering the actual winds at 500 mbs to be geostrophic, as is done in this study,  $\bar{u}_g = 0$ , and the poleward transport becomes <sup>1</sup>

$$M = \frac{2\pi a^2 \cos^2 \phi}{g} \int_0^{p_0} \overline{u_g v_g} \delta p \quad (9)$$

Using only the 500-mb data, the thickness  $\delta p$  is taken as one millibar, giving

$$M = \left( \frac{2\pi a^2 \cos^2 \phi}{g} \right) \overline{u_g v_g} \quad (10)$$

per millibar increment of pressure.

#### (b) The Fourier analysis

Any real, single-valued function of longitude  $f(\lambda)$ , at a particular latitude  $\phi$  and pressure  $p$ , may be expressed in terms of a Fourier expansion (after Saltzman [5, 6]) of the complex form

$$f(\lambda) = \sum_{-\infty}^{+\infty} F(n) e^{in\lambda} \quad (11)$$

where the complex coefficients,  $F(n)$ , are the Fourier transforms

$$F(n) = \frac{1}{2\pi} \int_0^{2\pi} f(\lambda) e^{-in\lambda} d\lambda \quad (12)$$

1. It may be shown that  $\overline{u'v'} = \overline{u_g v_g}$  if the geostrophic approximation is employed [1].



and  $n$  is any wave number.  $f(\lambda)$  and  $F(n)$  are called Fourier transform pairs. In this study, the following symbolism for transform pairs is used:

$u_g$	the eastward component of $\underline{V}_g$	$U(n)$
$v_g$	the northward component of $\underline{V}_g$	$V(n)$
$z$	the contour height	$A(n)$

Considering  $u_g$  and  $v_g$  we may write

$$\overline{u_g v_g} = \frac{1}{2\pi} \int_0^{2\pi} u_g v_g e^{-in\lambda} d\lambda \quad (13)$$

Applying Parseval's theorem, Saltzman [6] shows that

$$\overline{u_g v_g} = \sum_{-\infty}^{+\infty} U(n) V(n) \quad (14)$$

Expanding, and considering  $V(0) = 0$

$$\overline{u_g v_g} = \sum_n [U(n) V(-n) + U(-n) V(n)] \quad (15)$$

To obtain  $U(n)$ , we begin with the geostrophic approximation

$$u_g = -\frac{g}{f} \frac{\partial z}{\partial y} = -\frac{g}{fa} \frac{\partial z}{\partial \phi} \quad (16)$$



From the Fourier transform,

$$U(n) = \frac{1}{2\pi} \int_0^{2\pi} u_g e^{-in\lambda} d\lambda \quad (17)$$

Substituting (16) in (17),

$$U(n) = \frac{1}{2\pi} \int_0^{2\pi} -\frac{g}{fa} \frac{\partial z}{\partial \phi} e^{-in\lambda} d\lambda$$

Using  $z = \sum_{-\infty}^{\infty} A(n) e^{in\lambda}$ ;

$$U(n) = \frac{1}{2\pi} \int_0^{2\pi} -\frac{g}{fa} \left( \frac{\partial}{\partial \phi} \left[ \sum_{-\infty}^{\infty} A(n) e^{in\lambda} \right] \right) e^{-in\lambda} d\lambda$$

Since orthogonality applies,

$$U(n) = \frac{1}{2\pi} \int_0^{2\pi} -\frac{g}{fa} \frac{\partial A(n)}{\partial \phi} d\lambda = -\frac{g}{fa} \frac{\partial}{\partial \phi} A(n) \quad (18)$$

Similarly,

$$V(n) = \frac{ing}{fa \cos \phi} A(n) \quad (19)$$

The complex coefficient  $A(n)$  may be written  $C(n) e^{-in\psi(n)}$  where  $C(n)$  and  $\psi(n)$  are the real amplitude, and phase angle (in degrees of longitude), recorded in the Fourier analysis data [7]. These parameters henceforth are denoted  $C$  and  $\psi$ .

Substituting  $C e^{in\psi}$  for  $A(n)$  in Equations (18) and (19), and combining





terms yields

$$U(n)V(n) + U(-n)V(n) = \frac{2C^2 n g^2}{f a^2 \cos \phi} \frac{\partial \psi}{\partial \phi} \quad (20)$$

Applying the proper parameters for the particular latitude as found in Equation (10), an expression for the transport of angular momentum,  $J_M(n)$ , for wave number  $n$  is obtained:

$$J_M(n) = \frac{4\pi n^2 g \cos \phi}{f^2} C^2 \frac{\partial \psi}{\partial \phi} \quad (21)$$

#### 4. Computational and graphical procedure.

Equation (21) is adaptable to a form which can be used with a desk-calculator

$$J_M(n) = \frac{4\pi g \cos \phi}{f^2} n^2 C^2 \frac{\Delta \psi}{\Delta \phi} \quad (22)$$

$\frac{4\pi g \cos \phi}{f^2}$  is constant for a latitude and was computed for  $35^\circ$  N,  $45^\circ$  N,  $55^\circ$  N, and  $65^\circ$  N. The quantity  $n$  is the wave number under consideration,  $C$  is the amplitude of the Fourier wave component in feet as listed in the data for each latitude circle. In this study,  $\Delta \psi$  was taken as the difference in phase-angle over a  $10^\circ$  latitude-belt. For example, if  $J_M(n)$  is computed at  $35^\circ$  N,  $\Delta \psi_{35^\circ} = \psi_{40^\circ} - \psi_{30^\circ}$ , corresponding to a latitude-difference  $\Delta \phi = 10^\circ$ .

The meridional convergence of angular momentum transport is easily obtained from



$$\Delta J_M^{(n)} = J_M^{(n)}_{\phi_1} - J_M^{(n)}_{\phi_2} \quad (23)$$

Thus meridional convergence of angular momentum transport was computed for three ten-degree latitude belts,  $35^\circ$  N to  $45^\circ$  N,  $45^\circ$  N to  $55^\circ$  N, and  $55^\circ$  N to  $65^\circ$  N, for each wave number  $n = 1$  through 10, for each of the twenty days from 1 January 1948 to 20 January 1948.

Mean zonal wind speed for each ten-degree latitude belt was also computed, using the geostrophic approximation, for a twenty-five day period beginning 30 December 1947. A graph of these daily winds is shown in Fig. 3. The twenty-four hour changes of these zonal winds denoted  $\Delta u / \Delta t$  were computed and are shown in Fig. 4.

Correlation coefficients between daily values of  $\Delta J_M$  and  $\Delta u / \Delta t$  at the central latitude were desired, not only for the same day but for various time-lags on  $\Delta u / \Delta t$  also. The equation for the linear correlation coefficient

$$r = \frac{\sum_{i=1}^{20} (x_i - \bar{x})(y_i - \bar{y})}{20 s_x s_y} \quad (24)$$

was used in each latitude belt and for each individual wave number. An excellent correlation program devised by Ruggles [4] for use in the electronic computer NCR 102a was employed in this connection.

The mean transport of angular momentum for the twenty-day period was computed for waves  $n = 1-10$  for the four latitudes  $35^\circ$  N,  $45^\circ$  N,  $55^\circ$  N, and  $65^\circ$  N. The results of these computations are graphed in Fig. 5.



The mean meridional convergence of  $\overline{J}_M(n)$  for the twenty days was calculated for the three ten-degree latitude bands for waves  $n = 1-10$ .

Fig. 6 shows the results of these calculations. It is seen that there is a net convergence of angular momentum in all three latitude bands.

#### 5. Interpretation of the graphs.

According to Fig. 5, there is net northward angular momentum transport at  $35^\circ$  N,  $45^\circ$  N, and  $55^\circ$  N, but a negative transport at  $65^\circ$  N. This latter result is different from that of Saltzman[6], but is generally in keeping with the idea that angular momentum aloft should be transferred away from the region of surface easterlies. In the first three latitude groups, wave  $n=3$  accomplishes a sizeable fraction of the poleward transport. However there is a tendency for wave  $n=4$  to become more important than  $n=3$  in this respect as one proceeds from latitude  $35^\circ$  N to  $55^\circ$  N. In this same belt, wave  $n=5$  accomplishes equatorward transport. There is another maximum poleward transport accomplished by wave number group  $n=6-8$ , but primarily by wave  $n=6$ . The sign, or direction, of the transport depends upon the direction of the ridge (trough) relative to the meridian, and is governed entirely by the term  $\Delta\psi/\Delta\phi$  of Equation (22). Mean transports at  $n > 8$  become small and no conclusions will be drawn from this range.

Fig. 6 contains some of the key results of this paper: It establishes that for the three latitude belts involved, there was a net convergence of angular momentum. This is in keeping with the concept, demonstrated by





Mintz [3] and others, that angular momentum must be fed into the atmosphere above the surface westerlies. The wave group  $n=1$  to 3 provides strong convergence of momentum in each latitude belt, especially  $n=3$ , in the central latitude belt. The waves  $n=4$  or 5 provide sizeable divergence of angular momentum at latitudes  $40^\circ$  N and  $50^\circ$  N. Waves  $n=6$  and 7, however, appear to be operating together to bring about another maximum of convergence. It is difficult to extend these conclusions to the highest latitude belt where the distribution of convergence was relatively uniform with wave number. Finally, at waves  $n>7$ , the convergence values were generally much smaller than the values associated with wave groups just discussed.

## 6. Discussion of the correlations.

Correlation coefficients of  $\Delta J_M$  with  $\Delta u / \Delta t$  for each ten degree latitude band, for each wave number  $n=1$  through 10, and for five different lags applied to  $\Delta u / \Delta t$  were computed and are listed in Table 1. To evaluate the significance of the correlation coefficients computed, a method described by Hoel [p. 122, 2] was used. If a significance level of 0.05 is taken, it can be shown that an  $r$  of 0.44 is significant.

Examination of Table 1 shows seventeen significant correlation coefficients out of a total of one hundred and fifty, of which twelve are positive. The highest correlation coefficient computed was 0.60.

In Table 1 (a), where  $\Delta J_M$  lags  $\Delta u / \Delta t$  by 36 hours, there is only one significant  $r$ .





In Table 1 (b), which presents the correlations between  $\Delta J_M$  and  $\Delta u / \Delta t$  centered 12 hours earlier, there are five significant r's, all of which are positive. A positive correlation indicates that angular momentum is fed in or out of the belt in the same sense as the associated acceleration. In fact, out of a total of thirty r's in Table 1 (b), twenty are positive. This may indicate, in general, an overall small positive correlation between the meridional convergence of angular momentum flux  $\Delta J_M$  and  $\Delta u / \Delta t$  centered 12 hours before the convergence.

In the central latitude band of Table 1 (b), the two highest correlations (  $n=3$  and  $n=9$  ) correspond to sizeable net convergence values in Fig. 6. Apart from  $n=7$ , all the other correlations are extremely weak. In particular, note that for  $n=4$  and  $n=5$ , both waves yield sizeable values of convergence and divergence, respectively, but have no relationship to the expected value  $\Delta u / \Delta t$ . This seems to indicate that convergence in wave  $n=4$  is almost immediately compensated by divergence at  $n=5$ . Considering Fig. 6 together with Table 1 (b),  $\Delta J_M$  for  $n=3$  appears to be most closely linked to  $\Delta u / \Delta t$ .

In the lowest latitude band of Table 1 (b), waves  $n=2$  and 5 have significant correlations, and correspond to large values of 20-day mean convergence in Fig. 6. The largest contributor to net convergence,  $n=6$ , and the largest contributor to net divergence,  $n=4$ , both have low correlation coefficients, possibly indicating that these waves tend to neutralize each other.



In the high latitude band wave  $n=6$  shows a significant correlation and is a large contributor to net convergence.

Table 1 (c), in which  $\Delta u / \Delta t$  is centered 12 hours after  $\Delta J_M$ , shows three significant correlation coefficients, two of which are negative. It is difficult to draw conclusions from this table, except that  $\Delta J_M$  is becoming less reliable as an indicator of  $\Delta u / \Delta t$ .

Table 1 (d) with  $\Delta u / \Delta t$  centered 36 hours after  $\Delta J_M$  has two significant negative correlation coefficients, and two positive. It shows nineteen negative  $r$ 's out of a total of 30, almost a reversal of Table 1 (b). Rather than say that a convergence of angular momentum causes a decrease in wind speed about 36 hours later, it might be better to suggest that this result may be associated with the periodicity in the angular momentum convergence of the waves themselves, i.e., the time-change of  $C$  and  $\psi$  for a particular wave may be such that  $\Delta J_M (n)$  may have negative autocorrelation coefficients after some lag of, say, two days. A good example of this is wave  $n=1$ : followed through all of Table 1, at the low latitude band, the successive correlation coefficients spaced 24 hours apart are  $-.32$ ,  $-.35$ ,  $.00$ ,  $.49$  and  $.31$ .

Table 1 (e), with  $\Delta u / \Delta t$  centered 60 hours after  $\Delta J_M$ , shows four significant correlation coefficients. Again these correlations are probably a reflection of the periodicity of  $\Delta J_M$ .

Table 2 lists the contemporary correlation coefficients between  $\Delta J_M$  of waves  $n=1, 2$ , and  $3$ , and the other wave numbers up to  $n=10$ . Table 2 shows four significant  $r$ 's, and a lower percentage of significant correlation coefficients than is found in Table 1.



## 7. Conclusions.

In general, the correlation coefficients between  $\Delta J_M$  and  $\Delta u / \Delta t$  were low. The correlation coefficients with  $\Delta u / \Delta t$  centered 12 hours before  $\Delta J_M$  yielded the most significant correlations. High values of  $r$  with large lags of  $\Delta u / \Delta t$  are believed due to the periodicity of the individual waves, especially the periodic shifting of the higher latitude waves relative to the corresponding waves at lower latitudes.

Correlating the contemporary  $\Delta J_M$  of the very long waves  $n=1$ , 2, and 3 with the other waves did not yield conclusive results.

There was net convergence of  $J_M$  in all three latitude bands during the period of the study, which agrees well with Mintz [ 3 ] and others.

In the lower two latitude belts, waves  $n=1$ , 2, and 3 seemed to operate together to produce a major portion of convergence of  $J_M$ . Waves  $n=4$  and 5 appeared to produce net divergence of  $J_M$  and waves  $n=6$ , 7, and 8 produced a large portion of convergence of  $J_M$ . In the higher belt, all wave groups produce net convergence, the total being much less than in the lower latitudes, partly due to the shorter radius of the latitude circles.

As a result of this investigation, it appears that significant information may be revealed from an autocorrelation analysis of  $\Delta J_M$  with lags of 1, 2, 3 days, etc., for the individual waves and latitudes. One unknown factor in this study is the effect of "noise" in the Fourier



analysis. This arises as a result of error in the 500 mb analysis, the R.M.S. value of which has been found by the Joint Numerical Weather Prediction Unit to be 70 feet (verbal communication from CDR P. M. Wolff) on the basis of their 1977 point grid.







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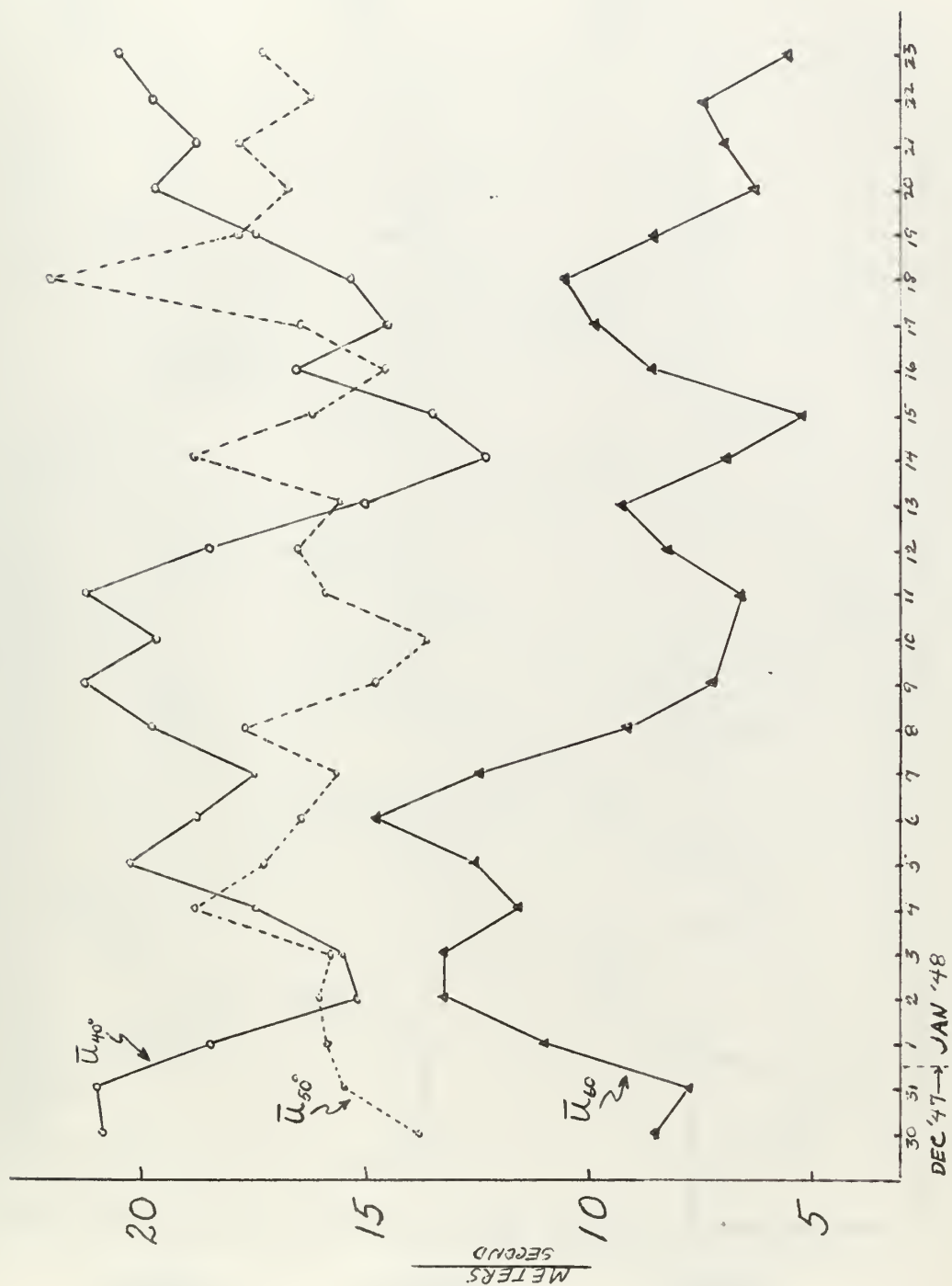


Figure 3. Daily values of the mean zonal wind across the three ten-degree latitude belts.



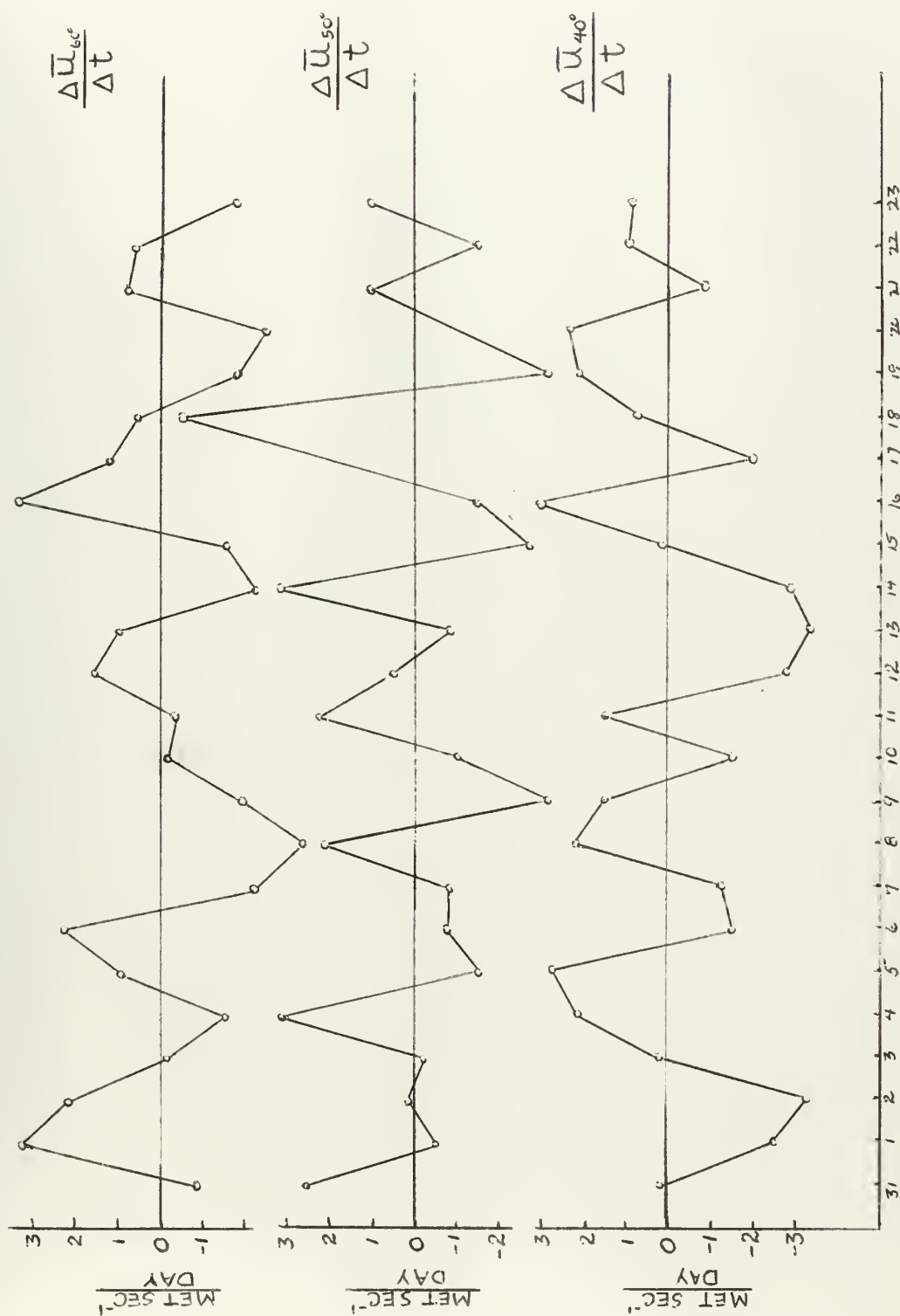


Figure 4. Daily change of the mean zonal winds across the three ten-degree latitude belts.



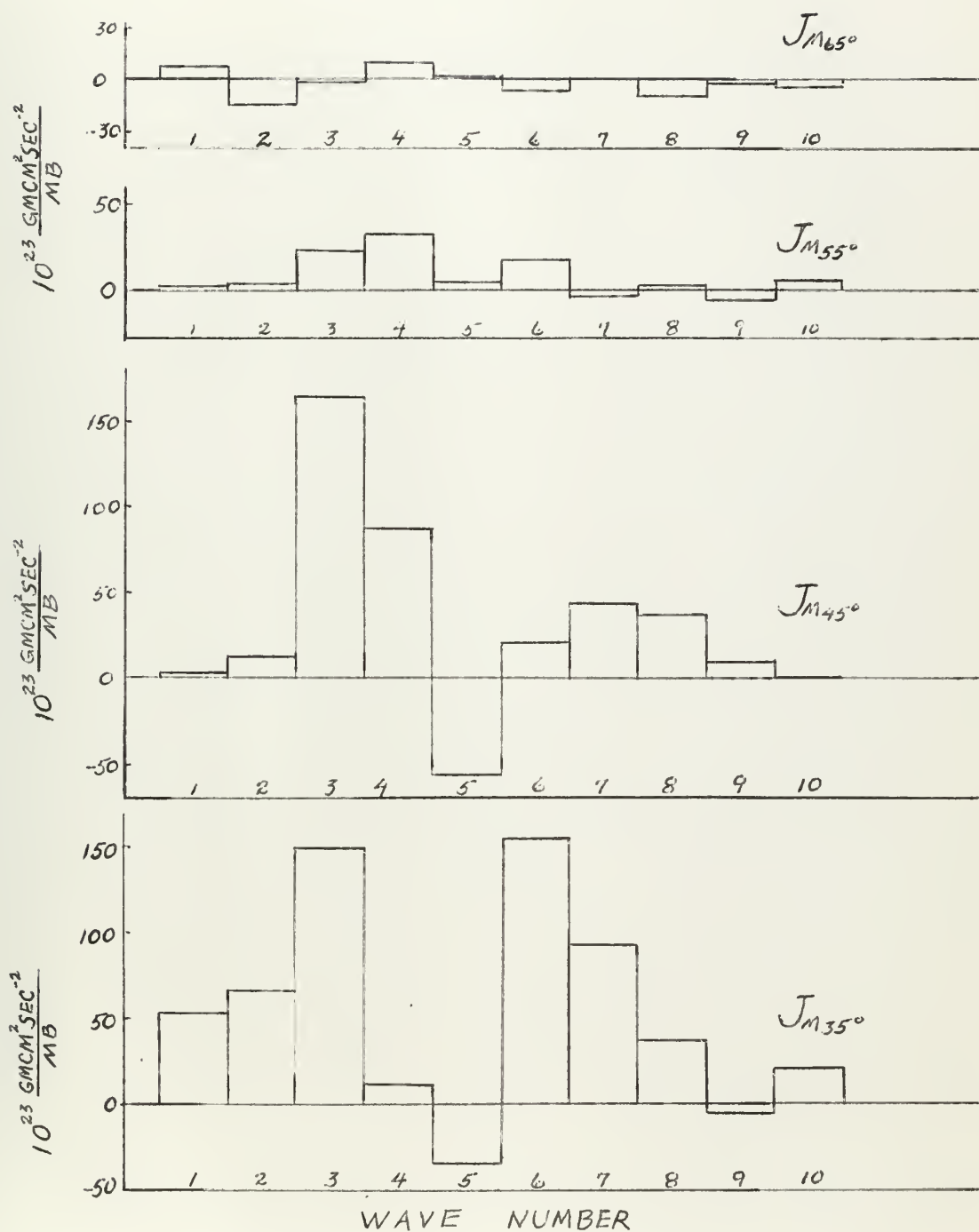


Figure 5. Mean (1-20 Jan) 500 mb geostrophic meridional momentum transport,  $J_M$ , across indicated latitude bands for waves  $n=1-10$ . Northward transport is indicated by positive sign.





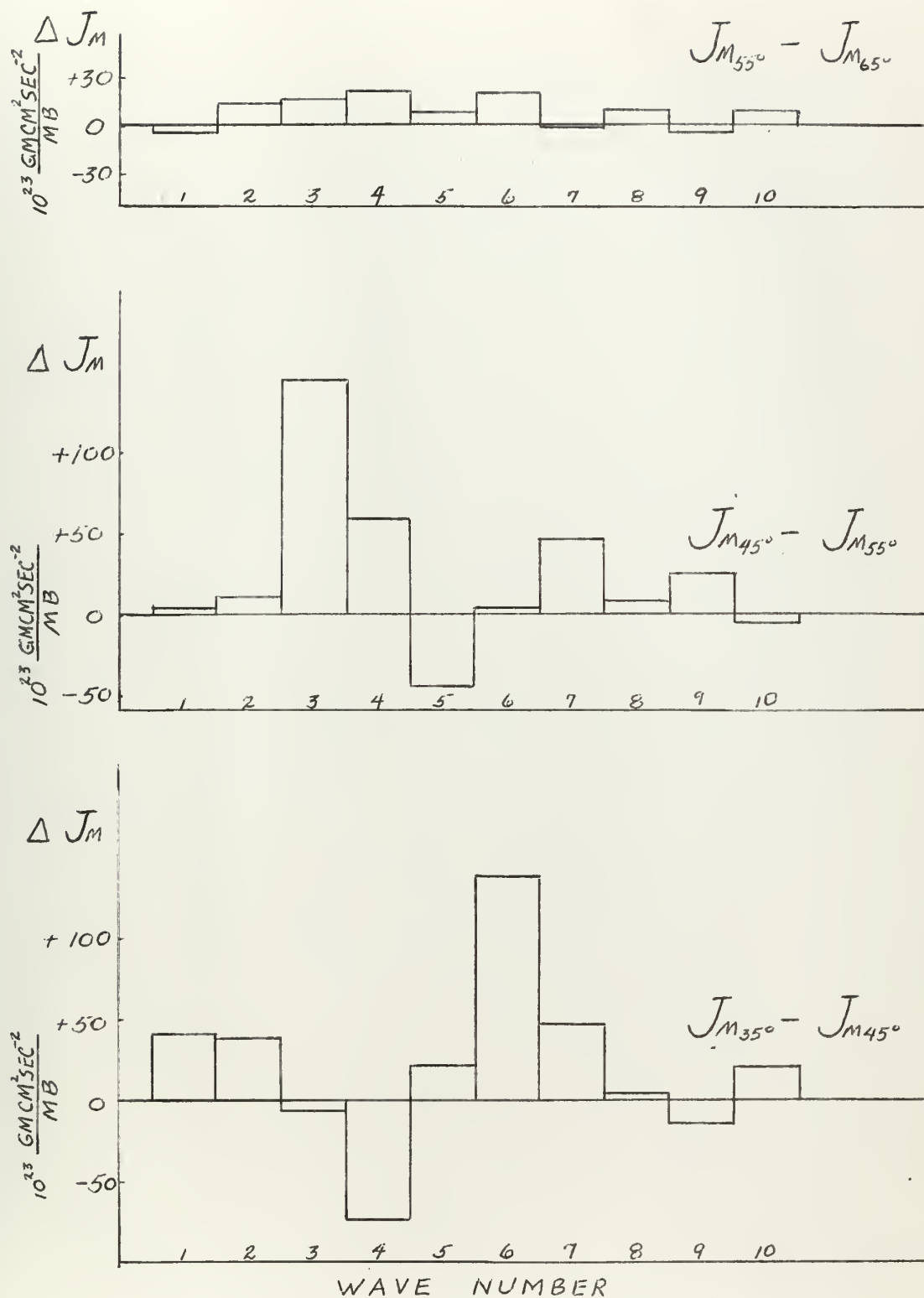


Figure 6. Mean values of the convergence of 500 mb geostrophic meridional momentum transport for the three ten-degree latitude belts indicated.



TABLE 1

Linear correlation coefficients  $\times 10^2$  between the meridional convergence of angular momentum transport and the change in zonal wind speed for  $10^\circ$  - latitude belt. The time lag of the change in wind speed is shown in each case.

(a) TIME LAG: $\frac{\Delta u}{\Delta t}$ centered 36 hours before $\Delta \bar{J}_M$										
n=	1	2	3	4	5	6	7	8	9	10
$\frac{\Delta u}{\Delta t}$	-32	-05	-17	-29	41	-30	30	04	41	16 $\Delta_{36} \bar{J}_M$
$\frac{\Delta u}{\Delta t}$	-13	-17	-14	41	-06	05	27	-20	00	-05 $\Delta_{41} \bar{J}_M$
$\frac{\Delta u}{\Delta t}$	-09	41	09	14	17	-12	-11	-06	19	51 $\Delta_{55} \bar{J}_M$
(b) TIME LAG: $\frac{\Delta u}{\Delta t}$ centered 12 hours before $\Delta \bar{J}_M$										
n=	1	2	3	4	5	6	7	8	9	10
$\frac{\Delta u}{\Delta t}$	-35	47	-06	17	50	-10	05	15	41	-11 $\Delta_{35} \bar{J}_M$
$\frac{\Delta u}{\Delta t}$	-08	-05	45	04	01	04	33	11	52	18 $\Delta_{45} \bar{J}_M$
$\frac{\Delta u}{\Delta t}$	-02	27	39	-21	-03	52	18	22	-12	09 $\Delta_{54} \bar{J}_M$



TABLE 1 (continued)

(c) TIME LAG:  $\frac{\Delta u}{\Delta t}$  centered 12 hours after  $\Delta \mathcal{J}_M$

n=	1	2	3	4	5	6	7	8	9	10
$\Delta u_{40}/\Delta t$	.00	-06	46	-19	13	33	22	-26	-02	10 $_{35}\Delta_{45}\mathcal{J}_M$
$\Delta u_{50}/\Delta t$	-29	11	14	-17	18	01	-44	42	-35	-38 $_{45}\Delta_{55}\mathcal{J}_M$
$\Delta u_{60}/\Delta t$	36	-08	20	08	-09	39	-06	-16	-20	-49 $_{55}\Delta_{65}\mathcal{J}_M$

(d) TIME LAG:  $\frac{\Delta u}{\Delta t}$  centered 36 hours after  $\Delta \mathcal{J}_M$

n=	1	2	3	4	5	6	7	8	9	10
$\Delta u_{40}/\Delta t$	49	28	12	-04	-16	32	-20	-09	-46	13 $_{35}\Delta_{45}\mathcal{J}_M$
$\Delta u_{50}/\Delta t$	30	-10	-29	-26	-17	-33	-26	-56	-15	44 $_{45}\Delta_{55}\mathcal{J}_M$
$\Delta u_{60}/\Delta t$	-00	02	-27	14	11	-15	16	-13	-05	-08 $_{55}\Delta_{65}\mathcal{J}_M$

(e) TIME LAG:  $\frac{\Delta u}{\Delta t}$  60 hours after  $\Delta \mathcal{J}_M$

n=	1	2	3	4	5	6	7	8	9	10
$\Delta u_{40}/\Delta t$	31	17	-26	17	13	-02	-15	32	24	-14 $_{35}\Delta_{45}\mathcal{J}_M$
$\Delta u_{50}/\Delta t$	16	32	16	-02	-03	26	60	12	-03	31 $_{45}\Delta_{55}\mathcal{J}_M$
$\Delta u_{60}/\Delta t$	-42	22	-47	-14	18	-06	-18	46	17	44 $_{55}\Delta_{65}\mathcal{J}_M$



TABLE 2

Contemporary linear correlation coefficients  $\times 10^2$  between the meridional convergence of angular momentum for listed wave numbers

(a)  $35^\circ$  N- $45^\circ$  N

n=	2	3	4	5	6	7	8	9	10
n=1	-04	-26	18	-11	-17	06	-05	-41	-02
n=2		19	16	15	24	-39	07	-04	14
n=3			-13	16	49	-44	-11	-21	37

(b)  $45^\circ$  N- $55^\circ$  N

n=	2	3	4	5	6	7	8	9	10
n=1	14	-43	-11	11	-06	21	-03	-04	06
n=2		-26	22	04	16	-13	13	24	-02
n=3			-26	-27	-08	27	24	-16	34

(c)  $55^\circ$  N- $65^\circ$  N

n=	2	3	4	5	6	7	8	9	10
n=1	07	-09	23	03	-24	-46	-37	35	-02
n=2		-05	-20	-24	27	19	-20	17	17
n=3			21	-57	15	35	-22	-34	-13







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